

$$y' = \frac{1}{1+yx^2}, \quad y(0) = -1$$

Integrate, let $2x = u$, then $2dx = du$,

$$\begin{aligned} \rightarrow y &= \frac{1}{2} \int \frac{2}{1+yx^2} dx = \frac{1}{2} \int \frac{1}{1+u^2} du = \frac{1}{2} \arctan u + C \\ &= \frac{1}{2} \arctan(2x) + C \end{aligned}$$

Apply initial condition:

$$y(0) = \frac{1}{2} \arctan(0) + C = -1$$

$$C = -1$$

so, $y = \frac{1}{2} \arctan(2x) - 1$