

$$\frac{dy}{dx} = y(1-y)$$

Separate:

$$\frac{1}{y(1-y)} dy = dx$$

Integrate:

$$\int \frac{1}{y(1-y)} dy = \int dx = x + C$$



$$\int \frac{1}{y(1-y)} dy, \text{ use P.F.D.}$$

$$\text{let } \frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{1-y}$$

$$\text{then, } 1 = A(1-y) + By$$

$$y=1 \rightarrow B=1$$

$$y=0 \rightarrow A=1$$

$$\text{so } \int \frac{1}{y(1-y)} dy =$$

$$\int \left(\frac{1}{y} + \frac{1}{1-y} \right) dy = \ln|y| - \ln|1-y|$$

$$\rightarrow \ln|y| - \ln|1-y| = x + c$$

$$\ln \left| \frac{y}{1-y} \right| = x + c$$

$$\frac{y}{1-y} = c e^x$$

$$y = c e^x - y c e^x$$

$$y + y c e^x = c e^x$$

$$y(1 + c e^x) = c e^x$$

$$y = \frac{c e^x}{1 + c e^x}$$

$$y = \frac{1}{c^{-1} e^{-x} + 1}$$