

$$\frac{dy}{dx} = 2y(3-y)$$

separate:

$$\frac{1}{2y(3-y)} dy = dx$$

Integrate:

$$\int \frac{1}{2y(3-y)} dy = \int dx$$



Use p.f.d. let

$$\frac{1}{2y(3-y)} = \frac{A}{2y} + \frac{B}{3-y}$$

$$\text{so, } 1 = A(3-y) + B(2y)$$

$$y=3 \rightarrow B = \frac{1}{6}$$

$$y=0 \rightarrow A = \frac{1}{3}$$

so,

$$\int \frac{1}{2y(3-y)} dy = \int \left(\frac{1}{3} \left(\frac{1}{2y} \right) + \frac{1}{6} \left(\frac{1}{3-y} \right) \right) dy$$

$$= \frac{1}{6} \ln|y| - \frac{1}{6} \ln|3-y| + C$$

$$\rightarrow \frac{1}{6} (\ln|y| - \ln|3-y|) = x + C$$

$$\ln|y| - \ln|3-y| = 6x + C_0$$

$$\ln\left|\frac{y}{3-y}\right| = 6x + C_0$$

$$\frac{y}{3-y} = C_1 e^{6x}$$

$$y = 3C_1 e^{6x} - yC_1 e^{6x}$$

$$y + yC_1 e^{6x} = 3C_1 e^{6x}$$

$$y(1 + C_1 e^{6x}) = 3C_1 e^{6x}$$

$$y = \frac{3C_1 e^{6x}}{1 + C_1 e^{6x}}$$

$$y = \frac{3}{C^{-1}e^{-6x} + 1}$$