

$$\frac{dy}{dx} = y(y-4)$$

Separate,

$$\frac{1}{y(y-4)} dy = dx$$

Integrate:

$$\int \frac{1}{y(y-4)} dy = \int dx$$

Use P.F.D.

$$\text{let } \frac{1}{y(y-4)} = \frac{A}{y} + \frac{B}{y-4}$$

$$\text{so, } 1 = A(y-4) + By$$

$$y=4 \rightarrow \frac{1}{4} = B$$

$$y=0 \rightarrow -\frac{1}{4} = A$$

$$\begin{aligned} \text{so, } \int \frac{1}{y(y-4)} dy &= \int \left(-\frac{1}{4} \left(\frac{1}{y} \right) + \frac{1}{4} \left(\frac{1}{y-4} \right) \right) dy \\ &= -\frac{1}{4} \ln|y| + \frac{1}{4} \ln|y-4| = x + c \end{aligned}$$

$$-\frac{1}{y} \ln|y| + \frac{1}{y} \ln|y-4| = x + C$$

$$-\ln|y| + \ln|y-4| = 4x + C_0$$

$$\ln \left| \frac{y-4}{y} \right| = 4x + C_0$$

$$\frac{y-4}{y} = C_1 e^{4x}$$

$$y-4 = y C_1 e^{4x}$$

$$y - y C_1 e^{4x} = 4$$

$$y(1 - C_1 e^{4x}) = 4$$

$$y = \frac{4}{1 - C_1 e^{4x}}$$